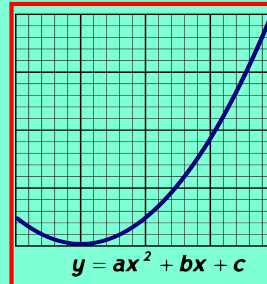


Math 125
Fall 2021
Lecture 39



Given $f(x) = \sqrt{6-3x}$

1) find $f(2) = \sqrt{6-3(2)} = \sqrt{6-6} = \sqrt{0} = 0$

2) find $f(-1) = \sqrt{6-3(-1)} = \sqrt{6+3} = \sqrt{9} = 3$

3) find its domain

Index = 2

Index = even

$(-\infty, 2]$

\Rightarrow Radicand ≥ 0

$$6-3x \geq 0$$

$$-3x \geq -6$$

$$\frac{-3x}{-3} \leq \frac{-6}{-3}$$

$$x \leq 2$$

4) find x such that $f(x) = 4$

Solve $f(x) = 4$

$$\sqrt{6-3x} = 4$$

$$(\sqrt{6-3x})^2 = (4)^2$$

$$6-3x = 16$$

$$-3x = 16-6$$

$$-3x = 10$$

$$x = \frac{-10}{3}$$

$\checkmark \left\{ \frac{-10}{3} \right\}$

Solve $\sqrt{x+5} - \sqrt{x} = 1$

$$\sqrt{x+5} = 1 + \sqrt{x}$$

$$(\sqrt{x+5})^2 = (1 + \sqrt{x})^2$$

$$x+5 = (1 + \sqrt{x})(1 + \sqrt{x})$$

$$x+5 = 1 + \sqrt{x} + \sqrt{x} + (\sqrt{x})^2$$

$$x+5 = 1 + 2\sqrt{x} + x$$

$$4 = 2\sqrt{x}$$

Divide by 2 to reduce

$$2 = \sqrt{x}$$

$$(2)^2 = (\sqrt{x})^2$$

$$\boxed{x=4} \quad \{4\}$$

check

$$\sqrt{4+5} - \sqrt{4} = 1 \rightarrow 1 = 1 \checkmark$$

$$\sqrt{9} - \sqrt{4} = 1$$

$$3 - 2 = 1$$

1) Simplify: $(2+3i)(4-i)$

$$= 8 - 2i + 12i - 3i^2$$

$$= 8 + 10i - 3(-1) = 8 + 10i + 3 = \boxed{11+10i}$$

2) Divide: $\frac{2+3i}{4-i} \cdot \frac{4+i}{4+i} = \frac{8+2i+12i+3i^2}{16+4i-4i-i^2}$

$$= \frac{8+14i-3}{16-(-1)} = \frac{5+14i}{17}$$

$$= \boxed{\frac{5}{17} + \frac{14}{17}i}$$

3) Rationalize the deno.: $\frac{1}{\sqrt{3}+\sqrt{2}}$

$$\frac{1}{\sqrt{3}+\sqrt{2}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{1(\sqrt{3}-\sqrt{2})}{\sqrt{9}-\sqrt{6}-\sqrt{6}-\sqrt{4}} = \frac{\sqrt{3}-\sqrt{2}}{3-2}$$

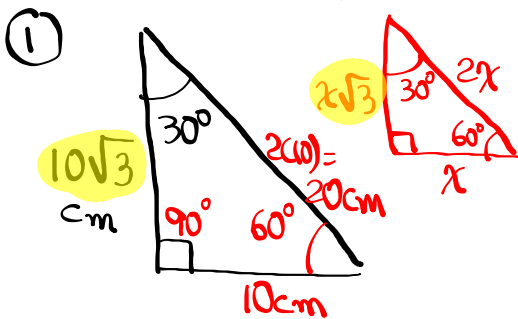
$$= \frac{\sqrt{3}-\sqrt{2}}{1} = \boxed{\sqrt{3}-\sqrt{2}}$$

1) Simplify: $i^{125} = i^{124} \cdot i = (i^2)^{62} \cdot i = (-1)^{62} \cdot i = 1 \cdot i = i$
 \boxed{i}

2) Simplify: $i^{27} = i^{26} \cdot i = (i^2)^{13} \cdot i = (-1)^{13} \cdot i = -1 \cdot i = -i$
 $\boxed{-i}$

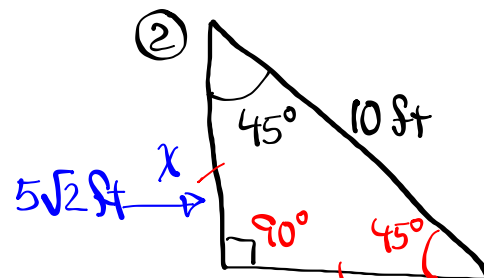
3) Simplify: $4\sqrt{250} - 3\sqrt{90} + 6\sqrt{40}$
 $= 4\sqrt{25}\sqrt{10} - 3\sqrt{9}\sqrt{10} + 6\sqrt{4}\sqrt{10}$
 $= 4 \cdot 5\sqrt{10} - 3 \cdot 3\sqrt{10} + 6 \cdot 2\sqrt{10}$
 $= 20\sqrt{10} - 9\sqrt{10} + 12\sqrt{10}$
 $\boxed{23\sqrt{10}}$

Find missing sides and missing angles:



$$x\sqrt{3} = 10\sqrt{3}$$

$$x = 10$$



$$x^2 + x^2 = 10^2$$

$$2x^2 = 100$$

$$x^2 = 50$$

$$x = \sqrt{50} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$$

Given $(x+3)^2 + (y-4)^2 = 4$
 $(x-h)^2 + (y-k)^2 = r^2$

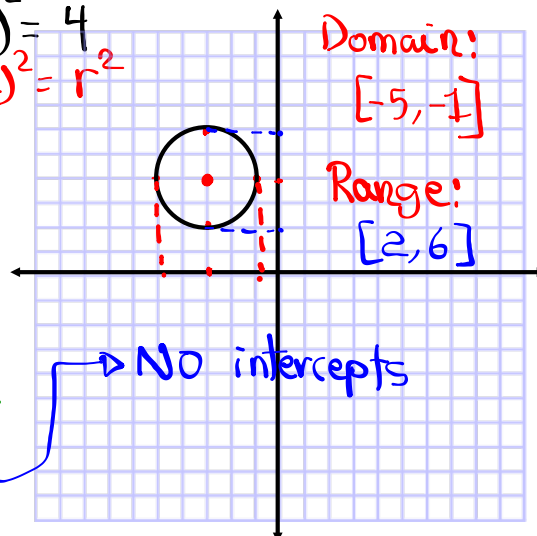
Center $(-3, 4)$

Radius 2

Draw

Discuss Domain & Range

Discuss all intercepts



Given $(x+5)^2 + (y+4)^2 = 16$

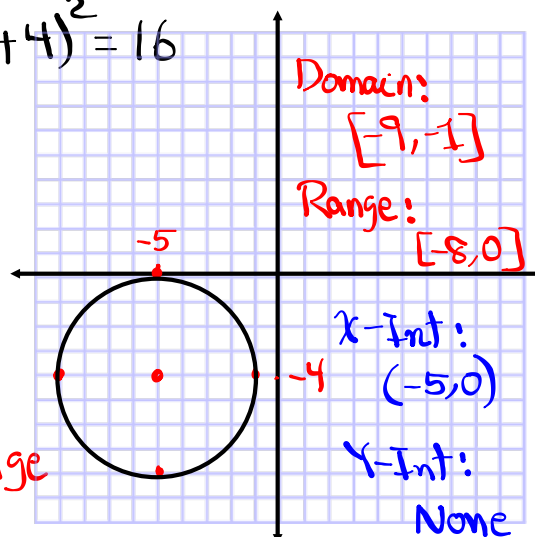
Center $(-5, -4)$

Radius 4

Draw

Discuss Domain & Range

Discuss all intercepts



Ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

\uparrow Always \uparrow Always

Center (h, k)

a units right & left to get additional pts.

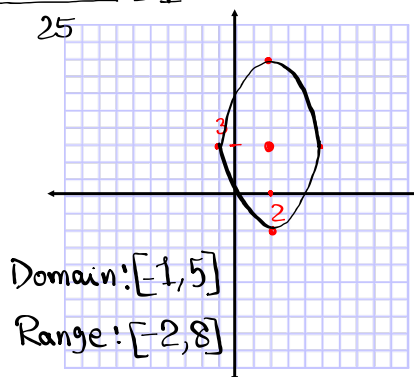
b units up & down

Ex: $\frac{(x-2)^2}{9} + \frac{(y-3)^2}{25} = 1$

Center $(2, 3)$

$a^2 = 9$ $a = 3$

$b^2 = 25$ $b = 5$



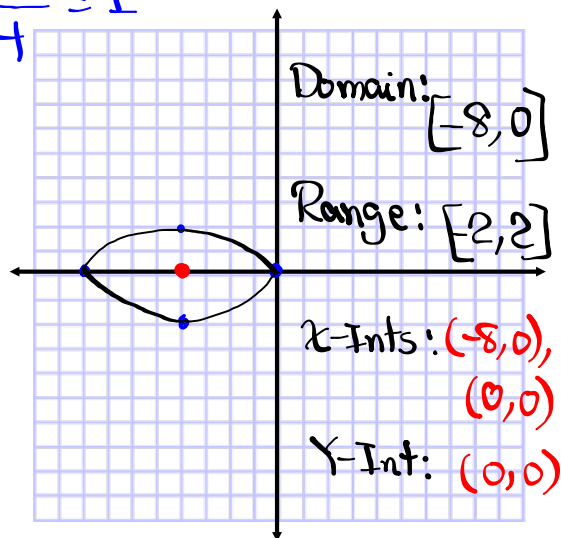
Given $\frac{(x+4)^2}{16} + \frac{y^2}{4} = 1$

Center $(-4, 0)$

$a^2 = 16$ $a = 4$

$b^2 = 4$ $b = 2$

Draw



$$25x^2 + 4y^2 = 100$$

Hint: Make
RHS 1.

Divide everything by 100

$$\frac{25x^2}{100} + \frac{4y^2}{100} = \frac{100}{100}$$

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

Center (0,0)

$$a^2 = 4 \quad a = 2$$

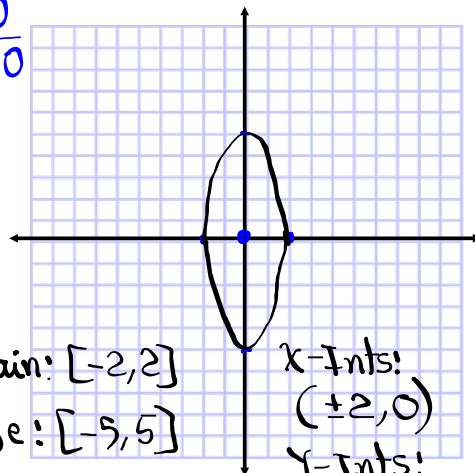
$$b^2 = 25 \quad b = 5$$

Domain: $[-2, 2]$

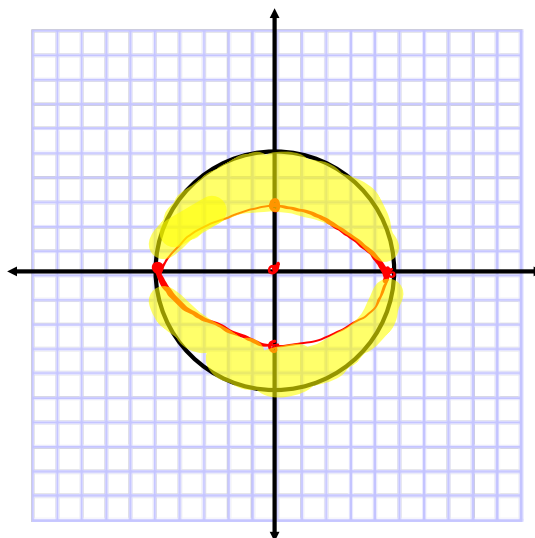
Range: $[-5, 5]$

x-Ints:
 $(\pm 2, 0)$

y-Ints:
 $(0, \pm 5)$



Graph $x^2 + y^2 = 25$ and $\frac{x^2}{25} + \frac{y^2}{9} = 1$,
then shade inside of circle and outside
of ellipse.



$$\text{Solve } \begin{cases} x^2 - 2y = 10 \\ -2 \begin{cases} 3x - y = 9 \end{cases} \end{cases} \Rightarrow \begin{cases} x^2 - 2y = 10 \\ -6x + 2y = -18 \end{cases}$$

$$\begin{array}{l} \text{when } x=4 \\ 3(4) - y = 9 \\ 12 - y = 9 \\ y = 3 \end{array} \quad \left. \begin{array}{l} \text{when } x=2 \\ 3(2) - y = 9 \\ 6 - y = 9 \\ y = -3 \end{array} \right\}$$

$$\boxed{(4, 3) \quad (2, -3)}$$

$$\begin{array}{r} x^2 - 6x = -8 \\ \hline \end{array}$$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x=4 & x=2 \end{array}$$

Solve

$$\begin{cases} 4x^2 + y^2 = 13 \\ -1 \begin{cases} x^2 + y^2 = 10 \end{cases} \end{cases} \Rightarrow \begin{cases} 4x^2 + y^2 = 13 \\ -x^2 - y^2 = -10 \end{cases}$$

$$\begin{array}{r} 3x^2 = 3 \\ \hline \end{array}$$

$$x^2 = 1 \quad \boxed{x = \pm 1}$$

$$1 + y^2 = 10$$

$$y^2 = 9$$

$$\boxed{y = \pm 3}$$

$$(1, 3), (-1, 3), (1, -3), (-1, -3)$$

Solve

$$\begin{cases} x^2 + 4y^2 = 16 \\ -1 \begin{cases} x^2 - y^2 = 1 \end{cases} \end{cases} \Rightarrow \begin{cases} x^2 + 4y^2 = 16 \\ -x^2 + y^2 = -1 \end{cases}$$

$$\hline 5y^2 = 15$$

$$y^2 = 3$$

$$\boxed{y = \pm\sqrt{3}}$$

$$x^2 - 3 = 1$$

$$x^2 = 4$$

$$\boxed{x = \pm 2}$$

$$(2, \sqrt{3}), (2, -\sqrt{3}), (-2, \sqrt{3}), (-2, -\sqrt{3})$$

Solve

$$\begin{cases} -4 \begin{cases} 3x^2 + 2y^2 = 35 \end{cases} \\ 3 \begin{cases} 4x^2 + 3y^2 = 48 \end{cases} \end{cases} \Rightarrow \begin{cases} -12x^2 - 8y^2 = -140 \\ 12x^2 + 9y^2 = 144 \end{cases}$$

$$y^2 = 4$$

$$\boxed{y = \pm 2}$$

$$4x^2 + 3(4) = 48$$

$$4x^2 + 12 = 48$$

$$4x^2 = 36$$

$$x^2 = 9$$

$$\boxed{x = \pm 3}$$

$$(3, 2), (3, -2)$$

$$(-3, 2), (-3, -2)$$